Building Quantum Computers

The global race to build the world's first quantum computer has attracted enormous investment from government and industry, and it attracts a growing pool of talent. As with many cutting-edge technologies, the optimal implementation is not yet settled. This important textbook describes four of the most advanced platforms for quantum computing: nuclear magnetic resonance, quantum optics, trapped ions, and superconducting systems. The fundamental physical concepts underpinning the practical implementation of quantum computing are reviewed, followed by a balanced analysis of the strengths and weaknesses inherent to each type of hardware. The text includes more than 80 carefully designed exercises with worked solutions available to instructors, applied problems from key scenarios, and suggestions for further reading, facilitating a practical and expansive learning experience. Suitable for senior undergraduate and graduate students in physics, engineering, and computer science, *Building Quantum Computers* is an invaluable resource for this emerging field.

Shayan Majidy is a Banting Fellow. He completed his Ph.D. at the Institute for Quantum Computing (IQC) at the University of Waterloo, Canada, where he earned prestigious awards including the Vanier Scholarship and IQC Achievement Award. He is recognized for his contributions in science education and outreach, founding a not-for-profit company to bring advanced science research to a wider audience. He has also taught quantum computing in various capacities, including as a sessional instructor, guest lecturer, and teaching assistant, and at multiple summer schools.

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Raymond Laflamme is a professor of physics at the University of Waterloo, Canada, and a pioneer in quantum computing. He was the founding director of the Institute for Quantum Computing (IQC), leading it from 2001 to 2017. Laflamme is renowned for his work on quantum error correction, the accuracy threshold theorem, the KLM model for photonic quantum computing, and experimental quantum information studies using nuclear magnetic resonance. He previously held the Canada Research Chair in Quantum Information and currently holds the Mike and Ophelia Lazaridis "John von Neumann" Chair in Quantum Information.

"*Building Quantum Computers* offers a concise yet comprehensive overview of quantum information processing and various cutting-edge technologies geared toward the development of quantum computers. I find this book particularly valuable due to its compendium-style presentation, making it an ideal resource for graduate students and for professionals actively involved in quantum computing research."
Rainer Blatt, Austrian Academy of Sciences

"*Building Quantum Computers* presents a pedagogical account of the basic quantum computing concepts and, perhaps for the first time, uniformly and comprehensively discusses the main hardware platforms that are pursued in building quantum computers. With carefully chosen end-of-chapter exercises, the authors provide an excellent textbook for advanced undergraduate and graduate students in physics and engineering who want to join the thriving quantum computation and simulation communities."
Pedram Roushan, Google

"In an age where it can be difficult to know who and what to trust, I am pleased to say you can 100% rely on these authors. They are not only renowned experts in their field, but they clearly worked very hard to make the difficult topics in this book not-so-difficult. It offers something that no other textbook does."
Olivia Lanes, IBM Quantum

"With its consistent notation, rigorous attention to detail, and a wealth of exercises, this textbook is an indispensable resource for students and researchers alike. Finally, the quantum information community has its pedagogical guide to modern quantum computing architectures."
Alexandre Blais, Université de Sherbrooke

"Quantum technology, at the intersection of physics, computer science, engineering, and mathematics, could very well be considered a discipline in its own right, and yet undergraduates rarely arrive at graduate-level quantum technology courses with enough background in each foundational discipline to be successful. A carefully crafted introduction to the topic of quantum technologies can make all the difference to one's success in this exciting and rapidly developing space, and this textbook delivers on this challenge. It serves as a lucid, accessible, and foundational introduction to the field of quantum technology, building upon tried-tested-and-true methods established over years at one of the world's most experienced quantum technology academic institutes."
Stephanie Simmons, Simon Fraser University

"*Building Quantum Computers* provides a very accessible treatment of the concepts underlying proposed implementations of quantum computers. It is a great resource to learn about the strengths, limitations, physical principles, and mathematical underpinnings of a variety of approaches to building and operating qubits."
Andrew Childs, University of Maryland

Building Quantum Computers

A Practical Introduction

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In memory of David Poulin, whose leadership, passion, and ability to explain quantum information to many audiences inspired us.

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Foreword

The principles of quantum mechanics, which as far as we know govern all natural phenomena, were discovered in 1925. For 99 years we have built on that achievement to reach a comprehensive understanding of much of the physical world, from molecules to materials to elementary particles and much more. No comparably revolutionary advance in fundamental science has occurred since 1925. But a new revolution is in the offing.

Up until now, most of what we have learned about the quantum world has resulted from considering the behaviour of individual particles – for example, a single electron propagating as a wave through a crystal, unfazed by barriers that seem to stand in its way. Understanding that single-particle physics has enabled us to explore nature in unprecedented ways, and to build information technologies that have profoundly transformed our lives.

What's happening now is we're learning how to instruct particles to evolve in coordinated ways that can't be accurately described in terms of the behaviour of one particle at a time. The particles, as we like to say, can become entangled. Many particles, like electrons or photons or atoms, when highly entangled, exhibit an extraordinary complexity that we can't capture with the most powerful of today's supercomputers, or with our current theories of how nature works. That opens extraordinary opportunities for new discoveries and new applications.

Most temptingly, we anticipate that by building and operating large-scale quantum computers, which control the evolution of very complex entangled quantum systems, we will be able to solve some computational problems that are far beyond the reach of today's digital computers. The concept of a quantum computer was proposed over 40 years ago, and the task of building quantum computing hardware has been pursued in earnest since the 1990s. After decades of steady progress, quantum information processors with hundreds of qubits have become feasible and are scientifically valuable. But we may need quantum processors with millions of qubits to realize practical applications of broad interest. There is still a long way to go.

Why is it taking so long? A conventional computer processes bits, where each bit could be, say, a switch which is either on or off. To build highly complex entangled quantum states, the fundamental information carrying component of a quantum computer must be what we call a "qubit" rather than a bit. The trouble is that qubits are much more fragile than bits – when a qubit interacts with its environment, the information it carries is irreversibly damaged, a process called decoherence. To perform reliable logical operations on qubits, we need to prevent decoherence by keeping the qubits nearly perfectly isolated from their environment. That's very hard to do. And because a qubit, unlike a bit, can change continuously, precisely controlling a qubit is a further challenge, even when decoherence is in check.

While theorists may find it convenient to regard a qubit (or a bit) as an abstract object, in an actual processor a qubit needs to be encoded in a particular physical system. There are many options. It might, for example, be encoded in a single atom which can be in either one of two long-lived internal states. Or the spin of a single atomic nucleus or electron which points either up or down along some axis. Or a single photon that occupies either one of two possible optical modes. These are all remarkable encodings, because the qubit resides in a very simple single quantum system, yet, thanks to technical advances over several decades, we have learned to control such qubits reasonably well. Alternatively, the qubit could be encoded in a more complex system, like a circuit conducting electricity without resistance at very low temperature. This is also remarkable, because although the qubit involves the collective motion of billions of pairs of electrons, we have learned to make it behave as though it were a single atom.

To run a quantum computer, we need to manipulate individual qubits and perform entangling operations on pairs of qubits. Once we can perform such single-qubit and two-qubit "quantum gates" with sufficient accuracy, and measure and initialize the qubits as well, then in principle we can perform any conceivable quantum computation by assembling sufficiently many qubits and executing sufficiently many gates.

It's a daunting engineering challenge to build and operate a quantum system of sufficient complexity to solve very hard computation problems. That systems engineering task and the potential practical applications of such a machine are both beyond the scope of *Building Quantum Computers*. Instead the focus is on the computer's elementary constituents for four different qubit modalities: nuclear spins, photons, trapped atomic ions, and superconducting circuits. Each type of qubit has its own fascinating story, told here expertly and with admirable clarity.

For each modality a crucial question must be addressed: how to produce well-controlled entangling interactions between two qubits. Answers vary. Spins have interactions that are always on, and can be "refocused" by applying suitable pulses. Photons hardly interact with one another at all, but such interactions can be mocked up using appropriate measurements. Because of their Coulomb repulsion, trapped ions have shared normal modes of vibration that can be manipulated to generate entanglement. Couplings and frequencies of superconducting qubits can be tuned to turn interactions on and off. The physics underlying each scheme is instructive, with valuable lessons for the quantum informationists to heed.

Various proposed quantum information processing platforms have characteristic strengths and weaknesses, which are clearly delineated in this book. For now it is important to pursue a variety of hardware approaches in parallel, because we don't know for sure which ones have the best long-term prospects. Furthermore, different qubit technologies might be best suited for different applications, or a hybrid of different technologies might be the best choice in some settings. The truth is that we are still in the early stages of developing quantum computing systems, and there is plenty of potential for surprises that could dramatically alter the outlook.

Building large-scale quantum computers is a grand challenge facing twenty-first-century science and technology. And we're just getting started. The qubits and quantum gates of the distant future may look very different from what is described in this book, but the authors have made wise choices in selecting material that is likely to have enduring value. Beyond that, the book is highly accessible and fun to read. As quantum technology grows ever more sophisticated, I expect the study and control of highly complex many-particle systems to become an increasingly central theme of physical science. If so, *Building Quantum Computers* will be treasured reading for years to come.

John Preskill
Pasadena, California

Preface

Fire to Qubits: The Cycles of Innovation

The great impetus behind scientific discovery is curiosity. When we encounter a puzzling observation, we formulate theories to explain it. These theories are refined or abandoned based on their agreement with further observations. Eventually, we achieve a level of understanding that allows us to control the phenomenon we observed, leading to the development of technologies that impact society. These technologies, in turn, raise new questions and challenges, enabling further observations that restart the cycle. It is this process that has distinguished humankind.

One of the earliest known examples of this cycle began with humanity's curiosity about fire. Initially, fire was a mysterious phenomenon that appeared when lightning struck a tree. Over time, humanity realized that combustion was not a random event. We discovered that striking stones together could produce sparks to ignite dry grass, and that pits could contain the resulting flames. Our control over fire led to the ability to cook food and melt metal to create tools. Initially, these tools made us more efficient at hunting and gathering, but eventually they caused a significant shift. Metal tools led to the development of agriculture and propelled humanity from living in nomadic tribes to settling in small villages.

Observing a more mundane phenomenon triggered another cycle 10 000 years later. This observation was that the lid of a kettle lifts when water boils. Understanding this commonplace occurrence was crucial for controlling steam, which led to the development of steam engines and the locomotive. The locomotive made it possible to travel much greater distances for resources. Ultimately, harnessing steam enabled cities to develop from villages.

In the same century, curiosity about electromagnetism brought the world closer together. First, in the 1820s, Hans Christian Ørsted linked electricity and magnetism by demonstrating that a compass needle moves in the presence of an electric current. Soon after, André-Marie Ampère discovered that parallel currents can attract or repel each other, and Michael Faraday proved that a moving magnet could generate a current. Finally, James Clerk Maxwell formulated a set of equations that unified these different effects. Maxwell's equations predicted the existence of electromagnetic waves, the understanding and control of which are crucial. Without this knowledge, many of today's technologies would be impossible. Curiosity about electromagnetic phenomena brought the world closer than even the locomotive could. In the last century, electromagnetism has raised questions about the nature of "information" itself. For example, wires become less reliable in transmitting information over long distances. This challenge led to the exploration of how to protect and compress information over such distances. The desire to process an increasing amount of information resulted in the development of computers. Through many such cycles, people around the world are now connected at their fingertips.

Reflecting on the curiosity of past generations raises an interesting question: What phenomena remain that, if harnessed, could have a comparable impact on society? The notion that there is nothing left to discover is a misleading trap that has ensnared even the most imaginative minds. In 1900, Lord Kelvin claimed, “There is nothing new to be discovered in physics now. All that remains is more and more precise measurement.” He added, “Neither the balloon, nor the aeroplane, nor the gliding machine will be a practical success.”

Only time can tell what the next great innovation cycle will be, but quantum technologies are showing tremendous potential. A little over a century ago, physicists encountered experimental results that were inexplicable with the physics of that time. Thus, a new model for describing nature emerged, one that could predict the behaviour of even single atoms. This marked the birth of the quantum era, leading to tools that can control atoms and the emergence of impactful quantum technologies. Examples of such technologies include medical imaging devices, lasers, and transistors. Yet, they harness only some of the potential offered by quantum mechanics, primarily utilizing quantized energy levels without tapping into the effect of quantum superposition. Other proposed quantum technologies, such as quantum cryptography and sensing, have also emerged. However, the most anticipated among these, the quantum computer, has yet to *fully* arrive. Researchers in both academia and industry have spent decades trying to build a practical quantum computer. After years of effort, quantum computers are now in a critical period of their development.

In the next decade, **noisy intermediate-scale quantum** (NISQ) computers are expected to become available. They are termed “noisy” because environmental effects limit the length of algorithms they can run, and “intermediate” due to their limited size. Researchers are developing NISQ-era computers using various physical systems. Despite their flaws, these computers may be leveraged for useful tasks. Some companies have already announced plans to make NISQ devices publicly available in the coming years. We may soon realize the long-touted potential of quantum computers. The same cycle that explored the characteristics of fire, steam, and electromagnetism is now leading us toward the era of quantum computing.

Motivation and Aim

The authors of this text have over three decades of collective experience teaching graduate courses, undergraduate courses, and summer schools on experimental quantum computing. These experiences, along with the steadily growing number of undergraduate and graduate courses in experimental quantum computing, have made it clear that a text on this topic would be useful.

Researchers in quantum information are interested in four broad questions:

1. What is quantum information?
2. What applications are possible with quantum information?
3. How can devices be built to implement these applications?
4. How can these devices be made robust and practical?

Different authors have written excellent texts on these first two questions. This text focuses on the third. The aim is not to describe the latest results, but to explain how we can use physical systems to manipulate information using quantum mechanics.

Content and Structure

We have organized this book into seven chapters. Chapter 1 begins with an overview of the history of quantum computers, outlining their rise from an obscure concept to an international endeavour. This chapter also introduces the criteria for building a functioning quantum computer and some fundamental themes of quantum computing. Chapter 2 briefly reviews the quantum mechanics necessary for understanding quantum computing. This chapter is expected to be a review for the reader, not their initial exposure to quantum mechanics. Chapters 3–6 explain how one of four quantum systems is used to build quantum computers. These systems are:

1. nuclear spins of a solution of molecules;
2. collection of photons;
3. ions in an electromagnetic trap; and
4. artificial atoms built from superconducting circuits.

These chapters discuss how we can use these different systems for quantum computing.[1] Researchers have proposed more systems for quantum computing than what we could fit in this text. We chose these four systems due to their technological maturity. Chapter 7 concludes with how we can benchmark the quality of different quantum computers.

Chapters 3–6 have the same structure. Each begins with a brief review of the physics involved in that implementation. They then explain how the quantum system in question satisfies each criterion for quantum computing. Next, the chapters discuss the sources of noise and each implementation's relative strengths and weaknesses. Many different techniques and approaches exist to perform quantum computations with each of these systems. These chapters don't attempt to survey them all. Instead, the focus is on understanding the underlying physics and some significant achievements in the respective fields.

Chapters 3–6 were written with the intention that a student would read them in order, and we recommend doing so. There are common threads between the chapters, and these chapters will occasionally refer back to a concept introduced earlier. However, Chapters 3–6 do not *directly* build off of one another. Thus, with some backtracking, Chapters 3–6 could be read independently.

Intended Use and Audience

We've written this book for a one-semester graduate or senior undergraduate course or a companion to a summer school on the physics of quantum computing. From our experience

[1] Some texts make a point of distinguishing between a "quantum information processor" and a "quantum computer." When authors make this distinction, a quantum information processor is typically understood as a machine that processes information encoded in quantum systems. A quantum computer is then a quantum information processor with no inherent scalability limit and can perform operations in a way that is robust to errors. We don't make this distinction in this text.

teaching this material to thousands of students and piloting this textbook for eight courses serving a few hundred students at the University of Waterloo, we are confident it is suitable for the following audiences. First, this text is not intended to be someone's first introduction to quantum computing, but a follow-up to an initial introduction to the theory of quantum information. Second, when used for a senior undergraduate course, the students should be physics majors. Lastly, we have used this textbook successfully to teach graduate students from physics, computer science, engineering, chemistry, and math backgrounds. This is made possible because (1) the book covers some basic concepts from various STEM fields not typically found in physics graduate textbooks; and (2), unlike undergraduate courses, the expectation with graduate courses should be that students will do some supplemental reading to fill gaps in their knowledge as needed. The authors of this book share a passion for bringing quantum computing to a broader cross section of society. Ideas are presented at a general and conceptual level whenever possible, before diving into the mathematical and technical details.

Pedagogical Features and Online Resources

This textbook contains the following pedagogical features:

- Chapters 2–7 conclude with selected exercises. These exercises range widely in their difficulty. Some can be solved with a few minutes of work, while others require more careful thought. These exercises are a tool for learners and instructors looking for assessment material.
- An online solutions manual has been made available for course instructors. The figure files have also been made available online for instructors.
- In each chapter, we provide a selection of review articles for further study. Chapters 2–6 also include references to textbooks containing a complete introduction to the background physics we review in the chapter.
- The four implementations this text covers typically use widely varying language and notation. We have presented all the implementations using a mostly common language and notation. Thus, despite the different fields of physics being addressed, each chapter reads as part of one cohesive text.
- Throughout the text, we introduce key quantum computing models and problems in their relevant settings. For example, we introduce the Rabi problem in NMR, different models of quantum computing in optics, and the Jaynes–Cummings model in superconducting.
- Each chapter begins with a detailed outline that provides the motivation for the context and the order in which it is presented.
- Chapters 3–7 end with summaries that round up the key points. Chapter 1 is too short to justify a summary, and Chapter 2 is already a summary itself.
- As explained above, we use the same structure for Chapters 3–6. Each of these chapters concludes with the relative strengths and weaknesses of each implementation to tie the implementations together.
- We have included a comprehensive list of the notation used in the text in the front matter.

Symbols

The notation presented here is introduced gradually throughout the textbook. This section is not meant to be precursory reading, and it is not expected that the reader will already be familiar with this notation. This page serves as a reference for the reader as they progress through the book. Unless otherwise stated, vectors and matrices are defined in the basis of the eigenstates of $\hat{\sigma}_z$. This book explores technologies from various areas of physics; to maintain consistency across different fields, the notation may occasionally change between chapters. It will be explicitly stated when such changes occur.

Review of quantum mechanics for quantum computing

$\|\psi\rangle = \alpha\|0\rangle + \beta\|1\rangle \equiv \begin{bmatrix}\alpha \\ \beta\end{bmatrix}$	Ket (one-qubit state)
$\langle\psi\| = \|\psi\rangle^\dagger = \alpha^*\langle 0\| + \beta^*\langle 1\| \equiv [\alpha^*, \beta^*]$	Bra (one-qubit state)
$\langle\phi\|\psi\rangle \equiv [\gamma^*, \delta^*]\begin{bmatrix}\alpha \\ \beta\end{bmatrix} = \gamma^*\alpha + \delta^*\beta$	Inner product
$\|\psi\rangle \otimes \|\phi\rangle = \begin{bmatrix}\alpha \\ \beta\end{bmatrix} \otimes \begin{bmatrix}\gamma \\ \delta\end{bmatrix} = \begin{bmatrix}\alpha\gamma \\ \alpha\delta \\ \beta\gamma \\ \beta\delta\end{bmatrix}$	Two-qubit state
$\hat{X}\|\psi\rangle = \|\phi\rangle$	An operator (denoted with a hat) acting on a state
$\|\psi\rangle\langle\phi\| = [\alpha^* \quad \beta^*] \otimes \begin{bmatrix}\gamma \\ \delta\end{bmatrix} = \begin{bmatrix}\alpha^*\gamma & \beta^*\gamma \\ \alpha^*\delta & \beta^*\delta\end{bmatrix}$	Outer product
$\hat{\sigma}_x = \begin{bmatrix}0 & 1 \\ 1 & 0\end{bmatrix}, \hat{\sigma}_y = \begin{bmatrix}0 & -i \\ i & 0\end{bmatrix}, \hat{\sigma}_z = \begin{bmatrix}1 & 0 \\ 0 & -1\end{bmatrix}$	Pauli matrices
$\mathbb{1} = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$	Identity operator
$\hat{\sigma}_+ = (\hat{\sigma}_x - i\hat{\sigma}_y)/2$	Raising operator
$\hat{\sigma}_- = (\hat{\sigma}_x + i\hat{\sigma}_y)/2$	Lowering operator
$\hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_j^{(2)} = \hat{\sigma}_i\hat{\sigma}_j$	Tensor product of operators
$\text{CNOT} = \begin{bmatrix}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0\end{bmatrix}$	The controlled NOT gate (matrix form)
$\text{CNOT} = \|0\rangle\langle 0\| \otimes \hat{\mathbb{1}}^{(2)} + \|1\rangle\langle 1\| \otimes \hat{\sigma}_x^{(2)}$	The controlled NOT gate (tensor product form)

$\mathrm{CZ} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$	The controlled $\hat{\sigma}_z$ gate
$\mathrm{CZ} = \|0\rangle\langle 0\| \otimes \hat{\mathbb{1}} + \|1\rangle\langle 1\| \, \hat{\sigma}_z.$	The controlled $\hat{\sigma}_z$ gate (tensor product form)
$\hat{A} = \hat{A}^\dagger$	Hermitian operator
$\hat{U}^\dagger = \hat{U}^{-1}$	Unitary operator
$\hbar = 1.054\,571\,817 \times 10^{-34} Js$	Planck's reduced constant
$\hat{H}$	Hamiltonian
$\langle \hat{A} \rangle_\psi = \langle \psi \| \hat{A} \| \psi \rangle$	Expectation value
$\rho = \|\psi\rangle\langle\psi\| = \begin{bmatrix} \|\alpha\|^2 & \alpha\beta^* \\ \alpha^*\beta & \|\beta\|^2 \end{bmatrix}$	Density matrix for a pure state
$\rho = \sum_i p_i \|\psi_i\rangle\langle\psi_i\|$	Density matrix for a state with probability p_i of being in the state $\|\psi_i\rangle$
$\mathrm{tr}\left[\hat{\rho}\right] = \sum_n \langle n\|\hat{\rho}\|n\rangle$	Trace, where $\{\|n\rangle\}$ is any set of basis states
$\mathrm{tr}_B[\hat{\rho}_{AB}] = \sum_m \langle b_m\|_B \, \hat{\rho}_{AB} \, \|b_m\rangle_B$	Partial trace of ρ_{AB} over B, where $\{\|b_m\rangle\}$ is a basis for B
$\vec{\hat{\sigma}} = [\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z]$	Pauli vector
$\left\|\psi(t_f)\right\rangle = \hat{U}(t_f, t_i) \, \|\psi(t_i)\rangle$	Time-evolved state
$i\hbar \frac{\mathrm{d}}{\mathrm{d}t} \|\psi(t_i)\rangle = \hat{H} \, \|\psi(t_i)\rangle$	The Schrödinger equation
$\hat{H} = -\frac{\hbar\omega}{2} \hat{\sigma}_z$	Typical form for a qubit's natural Hamiltonian
$\frac{\mathrm{d}}{\mathrm{d}t} \hat{A}^{(H)} = \frac{1}{i\hbar} \left[\hat{A}^{(H)}, \hat{H}\right]$	Heisenberg's equation
$\hat{R}(\vec{n}, \phi) = \exp\left(-i\frac{\phi}{2} \vec{n} \cdot \vec{\hat{\sigma}}\right)$	Rotation operator
$\hat{K}_j = \langle j\|^{(e)} \, \hat{U}(t) \, \|\psi\rangle^{(e)}$	Kraus operators
$p(0) = \mathrm{tr}\left[\hat{P}_0 \hat{\rho} \hat{P}_0\right] = \|\langle 0\|\psi\rangle\|^2$	The probabilities of observing the eigenvalue corresponding to the state $\|0\rangle$
$p(1) = \mathrm{tr}\left[\hat{P}_1 \hat{\rho} \hat{P}_1\right] = \|\langle 1\|\psi\rangle\|^2$	The probabilities of observing the eigenvalue corresponding to the state $\|1\rangle$
$\left[\hat{X}, \hat{Y}\right] = \hat{X}\hat{Y} - \hat{Y}\hat{X}$	Commutator
$\left[\hat{\sigma}_i, \hat{\sigma}_j\right] = 2i\epsilon_{ijk}\hat{\sigma}_k$	The Pauli matrices' commutation relation
ϵ_{ijk}	Levi-Civita symbol
$\Delta x \Delta p \geq \frac{\hbar}{2}$	The uncertainty principle between position x and momentum p
$\frac{\hat{p}^2}{2m} + \frac{m\omega^2\hat{x}^2}{2} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})$	Hamiltonian of quantum harmonic oscillator
$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}} \left(\hat{x} + \frac{i\hat{p}}{m\omega}\right)$	Annihilation operator
$\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}} \left(\hat{x} - \frac{i\hat{p}}{m\omega}\right)$	Creation operators

$\hat{N} = \hat{a}^\dagger \hat{a}$	Number operator
$\hat{a}\,\lvert\lambda\rangle = \lambda\,\lvert\lambda\rangle$	Definition of the coherent state $\lvert\lambda\rangle$
Nuclear magnetic resonance	
$\hat{S}_i = \frac{\hbar}{2}\hat{\sigma}_i$	Spin-$\frac{1}{2}$ operators (i = $\{x, y, z\}$)
$\hat{\vec{S}} := \hat{S}_x\vec{x} + \hat{S}_y\vec{y} + \hat{S}_z\vec{z}$	Total spin operator
γ	Gyromagnetic ratio
$\hat{\vec{\mu}} = \gamma\hbar\hat{\vec{\sigma}}/2$	Magnetic moment of nuclei
$\vec{B}$	Magnetic field
$\hat{H} = -\hat{\vec{\mu}}\cdot\vec{B}$	Hamiltonian of a nucleus with spin interacting with a magnetic field
δ_{CS}	Chemical shift
$\hat{H}_z = -\frac{\hbar}{2}(1-\delta_{CS})\gamma B_0\hat{\sigma}_z$	Hamiltonian of the spin-$\frac{1}{2}$ nuclei
$\omega_L = (1-\delta_{CS})\gamma B_0$	Larmor precession frequency of nuclei in constant magnetic field of strength B_0
J_{ij}	The J-coupling between the ith and jth nuclei
$\hat{H}_J = \frac{\hbar\pi J_{ij}}{2}\vec{\sigma}^{(i)}\cdot\vec{\sigma}^{(j)}$	Hamiltonian for the J-coupling between the ith and jth nuclei
$\hat{H}_{2q} = -\frac{\hbar\omega_{L1}}{2}\hat{\sigma}_z^{(1)} - \frac{\hbar\omega_{L2}}{2}\hat{\sigma}_z^{(2)} + \frac{\hbar\pi J_{12}}{2}\hat{\sigma}_z^{(1)}\hat{\sigma}_z^{(2)}$	General Hamiltonian for a two-qubit NMR sample, such as ^{13}C-chloroform
$\hat{U}_{\text{refocus}} = \hat{R}_x^{(1)}(-\pi)\hat{U}_J(t)\hat{R}_x^{(1)}(\pi)\hat{U}_J(t)$	The pulse sequence for refocusing a two-qubit system
$\langle M_x(t)\rangle = \text{Tr}\left[\hat{\rho}(t)(\hat{\sigma}_x^{(1)} + \hat{\sigma}_x^{(2)} + \cdots)\right]$	Expectation value of the nuclei's magnetization along the x-axis
$\hat{\rho}_{\text{th}} = \frac{e^{-\hat{H}/k_BT}}{\text{tr}\left[e^{-\hat{H}/k_BT}\right]} \approx \frac{1}{2^n}\left[\hat{\mathbb{1}} + \sum_i \frac{\hbar\omega_{L_i}}{2k_BT}\hat{\sigma}_z^{(i)}\right]$	Thermal state
$\hat{\rho}_{\text{PPS}} = \frac{1-\epsilon_n}{2^n}\hat{\mathbb{1}} + \epsilon_n \lvert 0\rangle\langle 0\rvert^{\otimes n}$	Pseudo-pure state
T_1	T_1 time, also known as the longitudinal relaxation time or thermal relaxation time
T_2	T_2 time, also known as the transversal relaxation time
T_2^*	The constant related to the dephasing we observe; pronounced T_2 "star"
Optics	
$\vec{\nabla}\cdot\vec{E}(\vec{r},t) = 0$	Gauss's law
$\vec{\nabla}\cdot\vec{B}(\vec{r},t) = 0$	Gauss's law for magnetism (in free space)
$\vec{\nabla}\times\vec{E}(\vec{r},t) + \frac{\partial\vec{B}(\vec{r},t)}{\partial t} = 0$	Faraday's law
$\vec{\nabla}\times\vec{B}(\vec{r},t) - \sqrt{\mu_0\epsilon_0}\frac{1}{c^2}\frac{\partial\vec{E}(\vec{r},t)}{\partial t} = 0$	Ampere's law (in free space)

$c = 2.99\,792\,458 \times 10^8$ m/s	Speed of light in a vacuum
$\nabla^2 \vec{A} = \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2}$	The wave equation (with wave speed c)
$\hat{H} = \sum_k \hbar\omega_k\left(\hat{a}_k^\dagger \hat{a}_k + \frac{1}{2}\right)$	Hamiltonian for the electromagnetic field
$\|0\rangle_k$	Vacuum state for the kth mode
$\|n\rangle_k = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}\|0\rangle_k$	Fock state for the kth mode
$\hat{X}_k = \frac{1}{\sqrt{2}}\left(\hat{a}_k + \hat{a}_k^\dagger\right)$	Amplitude quadrature
$\hat{P}_k = \frac{1}{i\sqrt{2}}\left(\hat{a}_k - \hat{a}_k^\dagger\right)$	Phase quadrature
$\|\alpha\rangle_k = \exp\left(-\frac{\|\alpha\|^2}{2}\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\|n\rangle_k$	Coherent state
$\|0\rangle_q := \hat{a}_2^\dagger\|00\rangle_{12} = \|01\rangle_{12}$	Dual rail encoding, logical 0
$\|1\rangle_q := \hat{a}_H^\dagger\|00\rangle_{VH} = \|01\rangle_{VH}$	Dual rail encoding, logical 1
$\|0\rangle_q := \hat{a}_2^\dagger\|00\rangle_{12} = \|01\rangle_{12}$	Polarization encoding, logical 0
$\|1\rangle_q := \hat{a}_V^\dagger\|00\rangle_{VH} = \|10\rangle_{VH}$	Polarization encoding, logical 1
$\hat{H}_{lo} = \hbar A_k \hat{a}_{k,\text{in}}^\dagger \hat{a}_{k,\text{in}}$	Hamiltonian of single-mode linear optical element
$\hat{H}_{lo} = \hbar\sum_{i,j=1}^{2} A_{k_i,k_j}\hat{a}_{k_i,\text{in}}^\dagger \hat{a}_{k_j,\text{in}}$	Hamiltonian of two-mode linear optical element
$\hat{H}_{\text{PS},k} = -\hbar\lambda_\phi \hat{a}_{k,\text{in}}^\dagger \hat{a}_{k,\text{in}}$	Hamiltonian of a phase shifter
$\hat{H}_{\text{BS},k_1k_2}(\kappa,\phi) = i\hbar\kappa\left(e^{i\phi}\hat{a}_{k_1,\text{in}}^\dagger\hat{a}_{k_2,\text{in}} - e^{-i\phi}\hat{a}_{k_2,\text{in}}^\dagger\hat{a}_{k_1,\text{in}}\right)$	Hamiltonian of a beam splitter
$v_{BS}(\theta,\phi) = \begin{bmatrix}\cos\theta & -e^{i\phi}\sin\theta\\ e^{-i\phi}\sin\theta & \cos\theta\end{bmatrix}$	The action of a beamsplitter in the notation introduced in Section 4.4.2
NS_{-1}	Nonlinear sign shift gate
$\|B_{ij}\rangle$	Bell basis, $ij = \{00, 01, 10, 11\}$
$\hat{D}_k(\alpha)$	Displacement operator
$\text{CZ}_{\frac{n^2}{(n+1)^2}}$	CZ gate which is performed with probability $\frac{n^2}{(n+1)^2}$
$\hat{S}_k(z)$	Squeezed operator
$\|z\rangle = \hat{S}_k(z)\|0\rangle = \exp\left([z^*\hat{a}_k^2 - z\hat{a}_k^{\dagger 2}]/2\right)\|0\rangle$	Squeezed state

Trapped ions

$\vec{F} = -k_x x\vec{x} - k_y y\vec{y} - k_z z\vec{z}$	Restoring force that is linear in the distance
U_0	The end cap electrode's static potential
$2d_0$	The distance between end cap electrodes
$V_0\cos(\Omega_m t)$	The parallel rod electrode's oscillating potential

s_0	The distance from the trap's central axis to the surface of the parallel rod electrodes
$\Phi_{EC} = \frac{U_0}{2d_0^2}(-x^2 - y^2 + 2z^2)$	Potential from the end cap electrodes
$\Phi_{PR}(x, y) = \frac{V_0}{2s_0^2}\cos(\Omega_m t)(s_0^2 + x^2 - y^2)$	Potential from the parallel rod electrodes
$\frac{\mathrm{d}^2}{\mathrm{d}\tau^2}x + [a - 2b\cos(2\tau)]x = 0$	Mathieu equation for x-axis
$\hat{x}_s$	Secular motion of ion
$\hat{x}_m$	Micromotion of ion
Ω_M	Angular frequency of Paul trap electrode
ω_z	Trap frequency
$\omega_r = \omega_y = \omega_x$	Radial frequency
ω_{CM}	Centre of mass frequency
ω_q	Ion resonance frequency
ω_l	Frequency of control laser
$\eta = k\sqrt{\frac{\hbar}{2m\omega_z}}$	Lamb–Dicke parameter
$\|n\rangle_M$	The motional state of the ion
$\hat{H}_{\text{car}} = \frac{\hbar\Omega}{2}\left(e^{i\phi_\ell}e^{-i\Delta t}\|e\rangle\langle g\| + h.c.\right)$	Hamiltonian for the carrier interaction
$\hat{H}_{\text{blue}} = \frac{\hbar\Omega}{2}\left(i\eta e^{i\phi_\ell}e^{-i(\Delta-\omega_z)t}\hat{a}^\dagger\|e\rangle\langle g\| + h.c.\right)$	Hamiltonian for the blue-sideband interaction
$\hat{H}_{\text{red}} = \frac{\hbar\Omega}{2}\left(i\eta e^{i\phi_\ell}e^{-i(\Delta+\omega_z)t}\hat{a}\|e\rangle\langle g\| + h.c.\right)$	Hamiltonian for the red-sideband interaction
$\hat{R}_{\text{car}}(\theta, \phi_\ell) = \exp\left[i\frac{\theta}{2}\left(\|e\rangle\langle g\| e^{i\phi_\ell} + \|g\rangle\langle e\| e^{-i\phi_\ell}\right)\right]$	Unitary evolution from a carrier interaction
$\hat{R}_{\text{blue}}(\theta, \phi_\ell) = \exp\left[i\frac{\theta}{2}\left(\hat{a}^\dagger\|e\rangle\langle g\| e^{i\phi_\ell} + \hat{a}\|g\rangle\langle e\| e^{-i\phi_\ell}\right)\right]$	Unitary evolution from a blue-sideband interaction
$\hat{R}_{\text{red}}(\theta, \phi_\ell) = \exp\left[i\frac{\theta}{2}\left(\hat{a}\|e\rangle\langle g\| e^{i\phi_\ell} + \hat{a}^\dagger\|g\rangle\langle e\| e^{-i\phi_\ell}\right)\right]$	Unitary evolution from a red-sideband interaction

Superconducting circuits

$\vec{J}$	Probability current
ρ	Probability density function
$\vec{v} = \frac{\hbar}{m}[\nabla\theta - \frac{q}{\hbar}\vec{A}]$	Condensate velocity
$\Phi_0 = \pi\hbar/e = 4.835\,978\,484 \times 10^{14}$ Hz/V	Flux quantum
V	Voltage
I	Current
Q	Charge
Φ	Flux
$L = V\left(\frac{\mathrm{d}I}{\mathrm{d}t}\right)^{-1} = \frac{\mathrm{d}\Phi}{\mathrm{d}I}$	Inductance
$C = I\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)^{-1} = \frac{\mathrm{d}Q}{\mathrm{d}V}$	Capacitance

$Q^2/2C$	Charging energy
$\Phi^2/2L$	Inductive energy
$\omega = \frac{1}{\sqrt{LC}}$	Frequency of LC circuit
$Z = \sqrt{\frac{L}{C}}$	Characteristic impedance of LC circuit
ϕ	Josephson phase
I_c	Critical current
$L_j = \Phi_0/(2\pi I_c)$	Magnitude of Josephson junctions inductance
$E_J = \Phi_0 I_c/2\pi$	Josephson energy
C_j	C of Josephson junction
C_s	Shunting capacitance
$C_\Sigma = C_j + C_S$	C of island
C_g	C of capacitor connected in parallel to Josephson junction
V_g	Potential from source connected to qubit
$\dot{\Phi}_g$	Flux across C_g
$\dot{\Phi}_j$	Flux across C_j
$\mathcal{L}$	Lagrangian
$\hat{n}$	Number of cooper pairs
$n_g = Q_g/2e = C_g V_g/2e$	Gate offset charge, or gate charge
$E_C = e^2/2(C_\Sigma + C_g)$	Charge energy
Φ_e	Flux-induced phase across the Josephson junction
ω_q	Qubit transition frequency
ω_r	Frequency of cavity
$g = e\omega_r\sqrt{\frac{Z_r}{2\hbar}}\frac{C_g}{C_\Sigma}\left(\frac{E_J}{2E_C}\right)^{1/4}$	Oscillator–transmon coupling constant or light–matter coupling constant
$\lvert g/e\rangle \otimes \lvert n\rangle$	Bare states
$\lvert +,n\rangle = \cos\left(\frac{\theta_n}{2}\right)\lvert e,n-1\rangle + \sin\left(\frac{\theta_n}{2}\right)\lvert g,n\rangle$	Dressed states
$\lvert -,n\rangle = -\sin\left(\frac{\theta_n}{2}\right)\lvert e,n-1\rangle + \cos\left(\frac{\theta_n}{2}\right)\lvert g,n\rangle$	
$\hat{H}_{JC} = \hbar\omega_r\hat{a}^\dagger\hat{a} + \frac{\hbar\omega_q}{2}\hat{\sigma}_z + \hbar g(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger)$	Jaynes–Cummings Hamiltonian
$\hat{N}_{tr}$	Operator that counts the total number of excitations
$\Lambda_j = \frac{jg^2}{\Delta-(j-1)E_C/\hbar}$	Lamb shifts
$\chi_j = \Lambda_j - \Lambda_{j+1}$	Dispersive shifts
$\omega_r' = \omega_r - \frac{\Lambda_2}{2} = \omega_r - \frac{g^2}{\Delta - E_C/\hbar}$	ω_r in dispersive regime
$\omega_q' = \omega_q + \Lambda_1 = \omega_q + \frac{g^2}{\Delta}$	ω_q in dispersive regime
$\chi = \Lambda_1 - \frac{\Lambda_2}{2} = \frac{g^2}{\Delta} - \frac{g^2}{\Delta - E_C/\hbar}$	Dispersive regime constant
ω_d	Frequency of the driving field

$\mathcal{E}(t)$	Amplitude of the driving field
ϕ_d	Phase of the driving field
$J = \frac{2E_{C1}E_{C2}}{\hbar E_{C_C}} \left(\frac{E_{J1}}{2E_{C1}}\right)^{\frac{1}{4}} \left(\frac{E_{J2}}{2E_{C2}}\right)^{\frac{1}{4}}$	Coupling strength between two transmons

Benchmarking

$F_s(\|\psi\rangle, \|\phi\rangle) = \|\langle\psi\|\phi\rangle\|^2$	State fidelity for pure states
$F_s(\rho, \sigma) = \mathrm{tr}\left[\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right]$	State fidelity for mixed states
F_e	Entanglement fidelity
r_Ψ	Error rate
V_Q	Quantum volume

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